Finding structures in the chaos of stratified turbulent flows

Dr Adrien Lefauve in collaboration with
Dr J. L. Partridge, Dr Q. Zhou, Prof C. P. Caulfield, Prof S. B. Dalziel, Prof P. F. Linden FRS
Department of Applied Mathematics and Theoretical Physics, University of Cambridge

Research topic
Turbulence in density-stratified fluids

- At high flow speeds, the interface is turbulent
- Complex, small-scale, unstable eddies
- Transport of salt / heat and momentum = mixing
- Mixing costs energy (‘tax’) and alters the flow

Applications
Predicting the mixing rate

- Natural ventilation of buildings
- Weather and climate simulations
- Pollutants dispersion and air quality

Challenges
Mathematical modelling

- The equations are well-known (Navier-Stokes)
\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} &\quad= \quad - \frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial x^2}, \\
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} &\quad= \quad 0, \\
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} &\quad= \quad 0, \\
\frac{\partial^2 \rho}{\partial x^2} &\quad= \quad 0,
\end{align*}
\]

- But… coupled nonlinear PDEs with many parameters: no exact solution ($\$1,000,000 prize!$)
- Need to develop simplified models using intuition from experiments

Experimental setup
Exchange flow through an inclined duct

- Sustains a stratified flow for long times
- Excellent model for many natural flows
- Control over key flow parameters
- Extrapolate to full size by dimensional analysis

Novel measurements
Velocity and density in a 3D volume!

- Cutting-edge technology pioneered in our lab

Data-driven model
Origin of this slow flow wave structure?

- Does it come from a flow instability?
- Combine 3D experimental data…

\[
\sigma^2 = \begin{bmatrix}
\rho_1 & \rho_2 & \rho_3 \\
\rho_2 & \rho_4 & \rho_5 \\
\rho_3 & \rho_5 & \rho_6
\end{bmatrix}
\]

- Solve a large numerical eigenvalue problem

Exciting results
Measured wave structure

- Predicted growing wave

Originality & Impact

Experiments
- Unprecedented measurements of complex velocity and density data in stratified flows
- Revealed puzzling slow structure: crucial building block for fast (turbulent) flows?

Mathematics
- Data-driven linear stability on 3D flows
- Explained the origin and properties of the slow structure (confined Holmboe mechanism)

Field data
- Possible relevance to estuarine flows

Find out more
(Use our code with smartphone camera and follow link)

STEM for Britain
Mathematical Sciences
March 9th 2020

UNIVERSITY OF CAMBRIDGE